A Complete Physical Frequency Dependent Lumped Model for RF Integrated Inductors

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***Abstract* — A new development in the modeling of planar inductors is presented. The inductance is evaluated by means of analytical expressions, taking into account a full geometry description and the frequency dependence. Moreover, the model predicts the behavior of the ohmic losses and the magnetically induced losses. Finally, a new electric model interpretation of the self-resonant frequency based on physical assumptions is given.**

I. Introduction

One of the limiting factors in the RFIC design has been the absence of high performance integrated passive components, mainly inductors and transformers. However, new techniques, like silicon Micromachining [1] or the use of high resistivity substrates, have been proved to increase the quality factor by removing the substrate losses. In those cases, metal losses become very important due to the ohmic and the magnetically induced losses and, then, copper interconnection becomes a good choice. The previous techniques can be classified as process techniques. More recently, there has been developed successfully new solutions, based on layout optimization [2], where the designer tries to find the optimum point between the ohmic and the magnetically induced losses.

Although it is stated that all these techniques have solved the lack of high performance components, there is still a major drawback: the lack of accurate lumped scalable models. Considerable efforts have been carried out and many of these models can be found in the literature [3]-[4]-[5]-[6].

However, all of them fail in describing important frequency phenomena:

1) In spite of their importance, magnetically induced losses in the substrate and in the metal strips are not considered and only ohmic losses are taken into account. Only fitting expressions, for particular cases are found in the literature [6].

2) The inductance value will depend on the Eddy current distribution and on the delay of the field through the structure and no insight has been done in this point.

3) The self-resonance of the component is normally modeled through the electrical couplings. Some efforts to calculated this value has been done using numerical methods [7] or using microstrip line theory [6], but it is normally left as a fitting parameter.

Another common problem is that models do not allow a full description of the inductor's geometry. For instance the width and the pitch of the planar inductor are fixed for all the turns; however, in layout optimization techniques the width and pitch of every turn becomes part of the design. Models that provide this facility can be found in [6]-[8].

Nowadays, on the other hand, electromagnetic simulators are very valuable tools: they can describe the behavior of a passive component with a very great degree of accuracy. With them, it is possible to study the physics of the previous phenomena. Therefore, in this work, we present an exhaustive analysis of inductors with the aid of such tools. The objective is to obtain a scalable physical lumped model. The model is based on the analysis of any turn to turn interaction of the coil. Moreover, it also accounts for the magnetically induced losses and gives a new insight in the calculation of the self-resonance of the inductor.

II. ELECTRICAL SCALABLE MODEL

Inductors for RFIC's design must have equivalent inductance values in the range of few nH, self resonant frequencies in the range of several GHz, and high values of the quality factor. All these requirements are hard to be fulfilled in standard silicon technologies. This is mainly due to the degrading effects related to the substrate. Therefore, silicon micromachining post-processing seems to be the best procedure to improve both, self-resonant frequency and quality factor. Once the substrate effects has been removed, the simplest model is the one composed by an inductor connected in series with a resistor and both in parallel with a capacitor.

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A. Inductor

In the microelectronic technologies, the common chosen shapes for planar inductors are the square and circular spirals. One way to compute their inductance value is through the well-known relation between the energy stored in the component and the intensity flowing in it:

$$E = \tfrac{1}{2} L |I|^2 \tag{1}$$

The energy can be calculated through the integration of the term $\mathbf{A} \cdot \mathbf{J}^*$ over the volume defined by the conductor, where **A** is the magnetic vector potential and **J** is the current distribution. Of course, using electromagnetic simulators, this can be accomplished for any shape of the metal strips, for example, the former ones.

When trying to explore analytical formulae, some sort of symmetry is needed to have a good accuracy. For example, in the case of circular spirals there still remains some sort of axial symmetry. Then it is possible to approximate the geometry to one based on concentric loops.

This last step can be done using two different criteria: (1) the perimeter of every circular loop must equal the perimeter of every turn of the spiral; (2) the area of every loop must be the same as the average area of every spiral turn. Choosing one or another depends on the geometry, basically on the value of the pitch. For instance, for small pitches, the first criterion would give a better description, while for large pitch it is better the second one. Based on these assumptions, it should be possible to analyse a circular spiral inductor with a 2-d axisymmetric model. The total energy of the system can be calculated through the next expression:

$$E = \frac{1}{2} \sum_{i,j} \int_{v_i} A_j \cdot J_i^* \, dv_i = \frac{1}{2} \sum_{i,j} E_{ij} \tag{2}$$

where the term E_{ij} is the energy associated to every pair of loops. In the case of the circular spirals, the integral evaluation can be solved analytically and the expression found is the next one:

$$E_{ij} = \mu \frac{I_j}{\ln\left(b_j / a_j \right)} \frac{I_i^*}{\ln\left(b_i / a_i \right)} \sum_{n=0}^{\infty} C_{2n+1} \, D_{2n+1} \tag{3}$$

where a_i and b_i are the inner and outer radius respectively of the loop **i**. The coefficients C_{2n+1} are found to be:

$$C_{2n+1} = \frac{n}{n+1} \left(\frac{2n-3}{n-1} - 1 \right) C_{n-2} \quad , C_1 = 1 \tag{4}$$

The coefficients D_{2n+1} are function of the current distribution in every loop. For a circular loop in the DC behavior they are:

$$D_{2n+1}^{i=j} = \frac{\left[r(2n+1) + a^{2n+2} / r^{2n+1} \right]_a^b + \left[r(2n+2) + r^{2n+2} / b^{2n+1} \right]_a^b}{(2n+2)(2n+1)}$$

$$D_{2n+1}^{i \neq j} = \frac{\left[r^{-(2n+1)} \right]_{b_j}^{a_j} \left[r^{(2n+2)} \right]_{a_i}^{b_i}}{(2n+2)(2n+1)} \tag{5}$$

The accuracy of these expressions has been corroborated with magnetic simulations performed with the ANSYS 5.6.2 solver for a family of inductors having a width of 20μm, a pitch of 40μm and an inner radius of 120μm. The results can be seen in the table I.

Notice that, with this model, it is possible to take into account the phase shift of the intensity between the loops and also describes the inductor in a full geometry sense: inner radius, width and pitch of every turn.

TABLE I
INDUCTANCE VALUE FOR CIRCULAR INDUCTORS

Turns	ANSYS	Analytical (3)
1	0.53	0.53
2	1.81	1.81
3	3.96	3.96
4	6.99	7.09

TABLE II
INDUCTANCE VALUE FOR SQUARE SPIRALS

Turns	ANSYS	MoM	Greenhouse	Analytical (6)
1	2.18	2.25	2.76	2.18
2	8.31	9.27	8.66	8.15
3	18.81	20.24	17.08	18.63
4	34.59	36.88	27.97	34.52

The former procedure can be applied with square inductors. In that case, it is possible to solve the term E_{ij} between two strips of metal [6]. It is also possible to find the inductance value by calculating the flux across the structure. To make this computation, it is necessary to find a function Φ_{ij} that relates the mutual flux between every pair of turns and then, to add all the possible contributions. This function is:

$$\begin{aligned} \Phi_{ij} = \frac{l_i \mu I}{\pi} \Bigg\{ & (r_j + 1) \left[0.8814 - \sinh^{-1}\left(\frac{r_j + 1}{|r_j - 1|} \right) \right] + \\ & + (1 - r_j) \left[(-1)^{i-j} \; 0.8814 - \sinh^{-1}\left(\frac{r_j - 1}{|r_j + 1|} \right) \right] + \\ & + \sqrt{2} \left[|r_j + 1| + |r_j - 1| \right] - 2 \sqrt{|r_j + 1|^2 + |r_j - 1|^2} \Bigg\} \end{aligned} \tag{6}$$

where $\boldsymbol{l_i}$ is the side length of turn i and r_j is the ratio defined by $(l_i - w_j/2) / l_j$, with $\boldsymbol{l_j}$ the side length of turn $\boldsymbol{j}$ and $\mathbf{w_j}$ its width Also, the formula has been tested with the electromagnetic simulator MoMemtum, with the magnetic simulator ANSYS and with the Greenhouse method, shown in the table II. Here, the geometry of the coil is a strip width of 100µm, a pitch of 200µm and an inner side of 900µm just to be compared with reference [4].

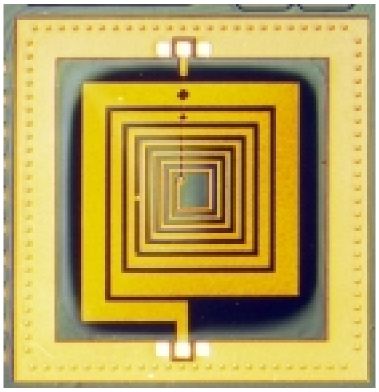

Fig. 1. Optimized inductor. Notice the different metal strip width for every turn (L = 32.25; Q = 17; SRF = 2.6 GHz).

Moreover, with the former expression it is possible to calculate the inductance value of the optimized layout inductors [4], as the one shown in the Fig. 1. The experimental value is 32.25 nH and the analytical one is 31.62 nH. It must be stressed that with other methods is not possible to predict that value for these structures.

B. Resistor

Regarding losses, the series resistance of the inductor model must account for losses due to conduction current (Ohmic losses) and losses due to magnetically induced currents (Eddy currents). Ohmic losses are directly related to the square resistance of the metal strip and the inductor's geometry. Losses due to Eddy currents are evaluated by the analysis of the current distribution along the inductor's structure. From this analysis, in the case of a constant width inductor, the frequency behavior of the series resistance can be expressed as follow:

$$R = R_{DC} + R_{RF}\frac{\left(f/f_o\right)^2}{\left(1+\left(f/f_o\right)^2\right)} \quad (7)$$

Where $\mathbf{R_{DC}}$ accounts for the ohmic losses, $\mathbf{R_{RF}}$ is related to the magnetically induced losses at very high frequency and $\mathbf{f_o}$ is a frequency factor that controls the transition from the low frequency behavior to the high frequency behavior. It should be noted that all the parameters from equation (7) are not fitting parameters: they are derived from physical assumptions [9]. Once more, electromagnetic simulators can be used to demonstrate the predicted behavior as it is shown in the Fig. 2.

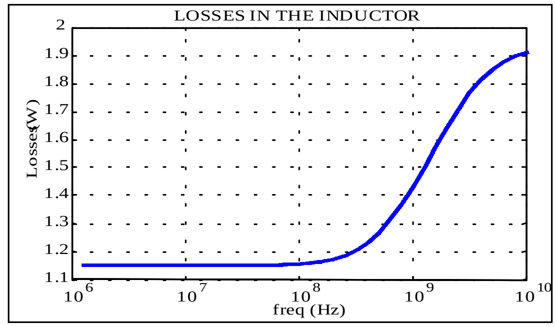

Fig. 2. Frequency behavior of the metal losses. It is possible to distinguish the transient governed by the fo parameter.

C. Capacitor

Regarding the capacitance of the inductors, some efforts have been done for calculating its value [7], but numerical techniques (for example, the moment method) are needed to compute the result. It collects the electrical coupling between both ports and, therefore, the dependencies on the technological parameters, basically dielectric constants and thickness of the substrate layers. This must be adjusted for each technology and the coupling can be divided in the electrical interaction of the bridge and the one between loops.

Normally, the capacitance value is set as a fitting parameter of the model to match the behavior of the structure near the self-resonance frequency. However, this method has some contradictions with physical facts. For example, when increasing the length of the inductor, the value of the capacitor must increase also. The Fig. 3 shows the value of the capacitance as a function of the number of strips in the inductor. Surprisingly, there are decreasing zones indicating that the capacitor value has no physical meaning.

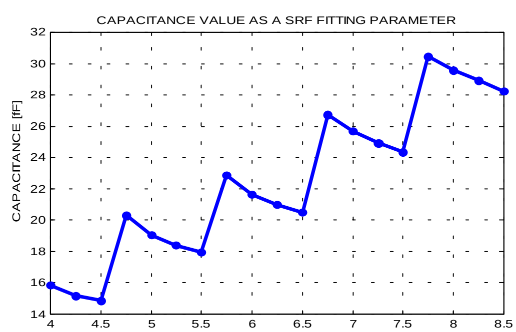

Fig. 3. Fitted value of the capacitance adjusted to self-resonant frequency as a function of the number of turns.

To explain this fact, let's suppose that there is a phase shift between the current in the turns. Therefore, the value of the inductance will decrease near the self-resonant frequency. So, if the DC value of the inductance is used in the model, then the fitting capacitor would underestimated the actual value.

This fact is explored in the model. With the aid of electromagnetic simulators, the behaviour of a loss-less inductor is calculated. Then, the equivalent inductance is compared with the one of an electrical model composed of an ideal inductor of value L_{DC} and a capacitor in parallel. The results are found in the Fig. 4, where the points indicate the ratio between the former two equivalent inductances. Moreover, on this plot the solid line shows the same ratio but calculated with the former analytical expression (6) when delay propagation is taken into account.

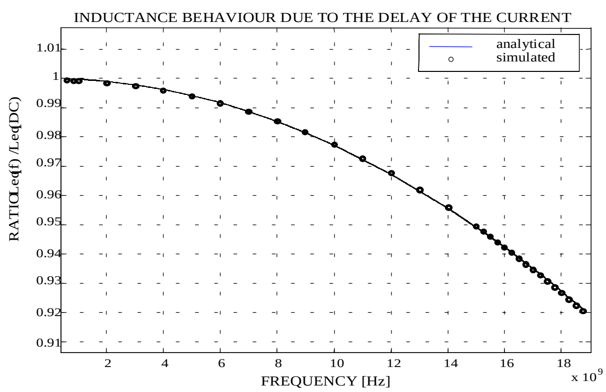

Fig. 4. Ratio between the equivalent inductance value calculated with an ideal inductor and calculated with MoMemtum (dots) or with expression (6) (solid line).

Finally, to test the accuracy of the developed model, Fig. 5 compares it with the experimental results.

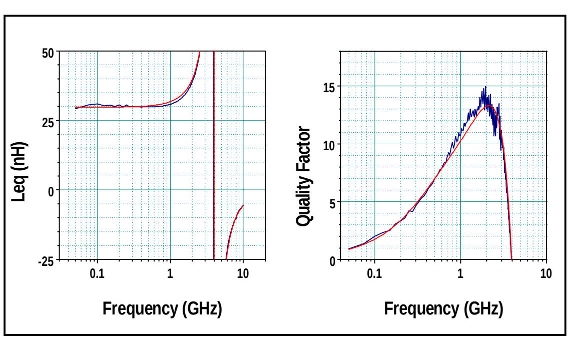

Fig. 5. Comparison between the model and the experimental results (geometry: w=51 µm, p=75 µm, inner side=110 µm and N=7).

III. Conclusions

This work presents relevant results of an exhaustive analysis of integrated inductors. The main features of the developed model are the ability to evaluate the equivalent inductance, including the phase shift in the current, and the frequency dependent series resistance (as a function of the strip width and pitch of the inductor's coil) by means of analytical expressions derived from physical considerations. Finally, the behavior near the self-resonant frequency is accurately described through capacitor values with physical meaning.

It has been also shown that electromagnetic simulators are a valuable tool for developing scalable models. The model can easily be implemented inside commercial design environments to insert high quality inductors in the design of oscillators, LNA and other MMIC-RFIC circuits.

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